B.Sc Part I, Physics (Hons)
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Q. What is D'Alembert's Principle?

Ans. **D'Alembert's Principle:→**

According to the principle of virtual work for a system to be in equilibrium, the resultant for acting on each particle of the system must be zero, i.e $\vec{F}=0$, where $\vec{F}_1$ is the force acting on the particle of the system. Then clearly the dot product of force $\vec{F}_1$ and the virtual displacement $\delta\vec{r}_1$ due to force $\vec{F}_1$ will be the virtual work must vanish

i.e $\vec{F}_1 \cdot \delta\vec{r}_1 =$ Virtual work $= 0$

∴ For the all particles of the particles of the system, we have

$$\sum \vec{F}_i \cdot \delta i = 0 \qquad \text{--- (I)}$$

This principle is also applicable to the statics of a system of principles, but it can be related to obtain a similar principle of dynemics.

According to Newton's second law of motion, the force is defined as the rate of change of momentum i.e

$$\vec{F}_1 = \frac{d\vec{P}_1}{dt} = \dot{\vec{P}}$$

where $\vec{P}_1$ is the momentum of ith particle due to Force $\vec{F}_1$ acting on the particle.

$$\vec{F}_i - \dot{\vec{P}}_i = 0 \qquad \text{--- (II)}$$

According to the above equation a moving system of particles can be considered to be in equilibrium under the force $(\vec{F}_i - \dot{\vec{P}}_i)$ i.e the actual applied $\vec{F}_i$ plus an additional Force $-\dot{\vec{P}}_i$ which is called reversed effective force of i^{th} particle.

Now replacing $\vec{F}_1$ of eq^n (I) by $(\vec{F}_i - \dot{\vec{P}}_i)$, we have

$$\sum_i (\vec{F}_i - \dot{\vec{P}}_i) \cdot \delta\vec{r}_i = 0 \qquad \text{--- (III)}$$

If the forces of constraints are present then, from

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the principle of virtual work

$$\vec{F}_i = \vec{F}_i^a + \vec{f}_i \quad \text{--- (IV)}$$

Here $\vec{F}_i^a$ is the actual force and $\vec{f}_i$ is the force of constraints acting on the i^{th} particle. Putting the value of $\vec{F}_i$ from eq^n (III) and (IV)

$$\sum_i \left\{ (\vec{F}_i^a + \vec{f}_i) - \dot{\vec{P}}_i \right\} \delta \vec{r}_i = 0 \quad \text{--- (V)}$$

or

$$\sum_i (\vec{F}_i^a - \dot{\vec{P}}_i) \cdot \delta \vec{r}_i + \sum_i \vec{f}_i \cdot \delta \vec{r}_i = 0 \quad \text{--- (VI)}$$

Again restricting ourselves to the case where the virtual work done by the forces of constraints vanishes, i.e

$$\sum_i \vec{f}_i \, \delta \vec{r}_i = 0$$

from eq^n (VI) reduces to

$$\sum_i (\vec{F}_i^a - \dot{\vec{P}}_i) \, \delta \vec{r}_i = 0 \quad \text{--- (VII)}$$

This equation is called D'Alembert's principle.

Since the virtual work done by the forces of constraints disappears, the superscript 'a' can be dropped in eq^n (VII) and the D'Alembert's principle can be written as

$$\boxed{\sum_i (\vec{F}_i - \dot{\vec{P}}_i) \cdot \delta_i = 0} \quad \text{--- (VIII)}$$